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An Average Based Method for Finding the Basic Feasible Solution for the Fuzzy Transportation Problems

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Abstract: Fuzzy transport problems are another special type of transport problem (TP). In a transportation problem, what is primarily considered is how to carry out the relevant process while reduce the total cost of the transporting the goods to different destinations. This objective is also valid for fuzzy TP. However, the supply quantity, demand, and unit cost values cannot be determined precisely, and those values are represented by "fuzzy number sets." There, the relevant solution value is obtained as a basic solution or an optimal solution. Thus, various researchers have proposed various algorithms to obtain an efficient initial solution or an optimal solution (OS) to fuzzy transportation problems. Accordingly, in this research article, we have presented another method to obtain an basic feasible solution (BFS) value for fuzzy transportation problems. It is prepared by creating a new value for each cell based on Yager's robust ranking method. In obtaining these values, the average of the crisp values of the columns and rows of the relevant column or row was basically considered. After that, the algorithm was used to solve mathematical problems. In here, the proposed method is primarily considered for triangular and trapezoidal fuzzy transportation problems. Also, the basic solution obtained from those solutions was that algorithm and the current approach are compared, and the efficiency and correctness of the proposed method were tested. Based on the analysis of the obtained data, the new method can be shown as an easy method to understand the efficiency that can be used to obtain the BFS to fuzzy transportation problems.

Keywords: Crisp Value, Fuzzy Transportation Problem, Initial Basic Solution, Optimal Solution, Robust Ranking Method

1. Introduction

The classic transportation problem (TP) is how to reduce the total cost of transporting commodities from different sources to different destinations. These problems are related in linear programming. F. L. Hitchcock [1] has first presented an algorithm to find the basic feasible solution (BFS) to this model. After that, various researchers have proposed new methods to obtain the basic solution as well as the optimal solution (OS) related to the above problems. Generally, in TP, the exact values of demand, supply, and unit cost of transportation per commodity are used. But the real-world situation doesn't allow for making a decision to use an exact value. Because problems are given fuzzy values [4], fuzzy concepts were introduced for linear programming in 1965 by Zadeh and Bellman [14]. Accordingly, the concept of fuzzy transportation problems was presented on the basis that it is not possible to present the above-mentioned values of a transportation problem. In fuzzy transmission problems, fuzzy values are set for demand and supply values [15]. The primary goal of the particular topic is to reduce the total fuzzy transmission cost.

In this research article, we aim to present the new algorithm for finding the BFS to the fuzzy transportation problem. Regarding the fuzzy number set, several researchers are focusing their work. There is a "Calculator for Fuzzy Numbers" published in 2019 [2], "Geometric mean method combined with ant colony optimization algorithm to solve multi objective problems in fuzzy environment " [3], "A Fuzzy Functional Network for Nonlinear Regression Problems [6]," and "The Theory of Triangular Fuzzy

Number," published by M. Clement Joe Anand in 2017.

In this research study, we mainly focus on finding the BFS for triangular and trapezoidal fuzzy transportation problems. Several research articles are published related to the triangular fuzzy transportation model. There are "An Approach for Solving Fuzzy Transportation" published in 2020 [7], "M. Deepa et al. presented "A New Approach to Solve Fully Fuzzy Transportation Problems Using Ranking Technique" [8], "A. Khoshnavaa and M. R. Mozaffarib" [9], and "On Solving Transportation Problems with Triangular Fuzzy Numbers: A Review with Some Extensions" published by Ali Ebrahimnejad in 2013 [10]. Trapezoidal fuzzy transportation problem related papers are "Solving Fuzzy Transportation Problem Using Trapezoidal Fuzzy Number" published by Dr. S. Akila and R. Raveena in 2022 [14], "A new algorithm for finding a fuzzy optimal solution for fuzzy transportation [16]", "New Approach for Solving Fuzzy Transportation Problem" presented by K. Nathiya et al. [17], "One Point Conventional Model to Optimize Trapezoidal Fuzzy Transportation Problem" by Dinesh C. S. Bisht et al. in 2019" by Dinesh C. S. Bisht et al. in 2019" by Dinesh C. S. Bisht et al. in 2019" by Dinesh C. S. Bisht e Also, papers are published on the following topics: Darunee Hunwisai and Poom Kumam in 2017 [19], "An Algorithm to Solve Fuzzy Trapezoidal Transshipment" presented by P. Gayathri and K. R. Subramanian [20], Rajshri Gupta et al. present the algorithm in 2017 [21], and "A Method for Solving Fuzzy Transportation Problem (FTP) Using Fuzzy Russell's Method" published by S. Narayanamoorthy et al. in 2013 [22].

Although this research only deals with triangular and trapezoidal fuzzy transportation problems, there are also numerical transport problems. For example, articles published under the titles "A naive algorithm to solve the pentagonal fuzzy transportation problem [11]" and "Geometric Mean with Pythagorean Fuzzy Transportation Problem [13]" can be pointed out. And transportation problems have been prepared using the lessons of real life. Fuzzy transportation problems are also applied in everyday life, and a related research paper has been presented by "A. Venkatesh and A. Britto Manoj" in 2019 [24].

In here, we propose an alternate algorithm for finding a BFS to the fuzzy triangular and trapezoidal transportation problem. Regarding the mentioned algorithm, it is basically based on the Yager's robust ranking [23] method with the proposed base method; also, that algorithm is compared with the existing method and focuses on the efficiency of the proposed method.

2. Methodology

2.1. Preliminaries

Mathematically, the fuzzy transportation problem can be represent using the subsequent table, and each element in the table can be described as follows.

Table 1. *Fuzzy transportation table.*

	D_1	D_2	…	D_n	Fuzzy supply α_i
S_1	$\tilde{C}_{11}$ X_{11}	$\tilde{C}_{12}$ X_{12}	…	$\tilde{C}_{1n}$ X_{1n}	α_1
S_2	$\tilde{C}_{21}$ X_{21}	$\tilde{C}_{22}$ X_{22}	…	$\tilde{C}_{2n}$ X_{2n}	α_2
⋮	⋮	⋮	⋮	⋮	⋮
S_m	$\tilde{C}_{m1}$ X_{m1}	$\tilde{C}_{m2}X_{m2}$	…	$\tilde{C}_{mn}$ X_{mn}	α_m
Fuzzy demnd β_j	β_1	β_2	…	β_n	$\sum_{i=1}^{m}\alpha_i = \sum_{j=1}^{n}\beta_j$

$\tilde{C}_{ij}$- The cost of moving a unit from i[th] source to the j[th] destination which is fuzzy transportation.

X_{ij} -quantity of units moved from the source (i) to the destination (j).

$\alpha_i -$ available commodities in i[th] sources.

β_j – number of commodities requirements in j[th] destination

Objective in this algorithm is find the value of Minimize$Z = total\ fuzzy\ cost\ \sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}C_{ij}$

In here

When $\sum_{i=1}^{m}\alpha_i = \sum_{j=1}^{n}\beta_j$ problem is balance fuzzy transportation and $\sum_{i=1}^{m}\alpha_i \neq \sum_{j=1}^{n}\beta_j$ problem is unbalance fuzzy TP.

The main goal in this study is to find the BFS for transportation problem by using alternate method. the basic solution is called when the number of assignments given I the fuzzy transportation problem is equal to "m+n-1". When the answer is obtaining the minimum value, it is called the optimum solution.

We use the Yager's Robust ranking technique to solve fuzzy transportation problems. Before the discuss fuzzy concepts, we need some definitions studied.

Definition 2.1.1 Fuzzy set – A fuzzy set Ã of Ṛ (real number) is fuzzy number if it is said membership function $\mu_{\tilde{A}}$(Ṛ) [0,1] and satisfy the following properties. [16]

Ã must be a normal fuzzy set and $\boldsymbol{X}\ \boldsymbol{\mathcal{E}}$ Ṛ such that $\mu_{\tilde{A}}(\vec{x})$ =1

Ã is convex, $X\ \mathcal{E}$ Ṛ

$\mu_{\tilde{A}}$ is upper semi continuous

Sup[Ã] is bounded in Ṛ

Definition 2.1.2 Triangular fuzzy numbers [5] – ã= (a_1, a_2, a_3) is triangular fuzzy number with membership function is mention follows;

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & a_1 \leq x \leq a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & a_2 \leq x \leq a_3 \\ 0\ else & \end{cases}$$

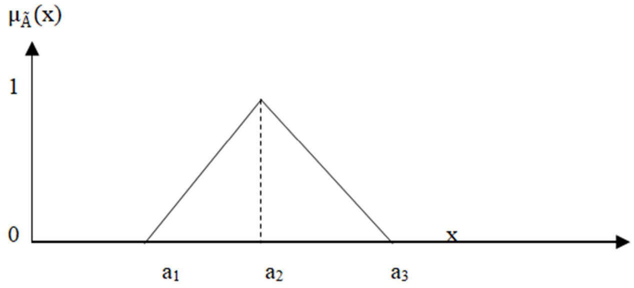

Figure 1. *Presentation of the triangular fuzzy numbers.*

Definition 2.1.3 trapezoidal fuzzy number [12]- ã= (a_1, a_2, a_3, a_4) is triangular fuzzy number with membership function is mention follows;

$$\mu_{\tilde{A}}(x) \begin{cases} 0 \; x \le a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} \; a_1 \le x \le a_2 \\ 1 \; a_2 \le x \le a_3 \\ \frac{(a_4-x)}{(a_4-a_3)} \; a_3 \le x \le a_4 \\ 0 \; else \end{cases}$$

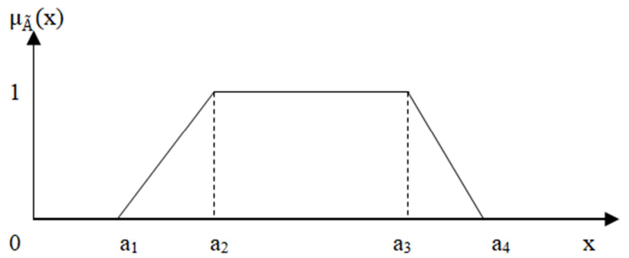

Figure 2. *Presentation of the trapezoidal fuzzy number.*

The following definition can be shown as an important property in the fuzzy transportation problem study.

Additional property of two fuzzy membership function { ã= (a_1, a_2, a_3) and $\tilde{b}$ = (b_1, b_2, b_3) }

$$\tilde{a} + \tilde{b} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1+ b_1, a_2+ b_2, a_3+ b_3)$$

In this study, the algorithm has been developed based on Yager's ranking method. [23]

Definition 2.1.4 Yager's ranking method – let take the fuzzy number is ã, then Yager's ranking index values for;

1) Triangular fuzzy set

$$R(\tilde{a})=\int_0^1 0.5(a_\alpha^l \, , a_\alpha^u)d\alpha$$

Where (a_α^l, a_α^u) is α cut level of the fuzzy number ã

$$a_\alpha^l = (a_2 - a_1)\alpha + a_1$$

$$a_\alpha^u = (a_3 - a_2)\alpha + a_3$$

2) Trapezoidal fuzzy set

$$R(\tilde{a})=\int_0^1 0.5(a_\alpha^l \, , a_\alpha^u)d\alpha$$

Where (a_α^l, a_α^u) is α cut level of the fuzzy number ã

$$a_\alpha^l = (a_2 - a_1)\alpha + a_1$$

$$a_\alpha^u = (a_4 - a_3)\alpha + a_4$$

2.2. Proposed Algorithm

Step 1. First check fuzzy TP is balanced or unbalanced, and if unbalanced, balance it with a dummy column or row as appropriate.

Step 2. After that, calculate the crisp values related to each cell as well as supply and demand by Yager's Robust method.

Step3. Then calculate the mean of the corresponding column and row values. Name the mean of the columns $(A.C)_i$ and the mean of the rows $(A.R)_j$.

Step 4. Then get the new value of each cell's crisp value from the sum of the average values of the corresponding column and row by the following equation:

$$V_{ij} = \frac{(A.R)_j + (A.C)_i}{2} - \tilde{C}_{ij}$$

Step 5. Appropriately assign the corresponding assignment values to the corresponding cells in order of decreasing new value.

Step 6. Given the assignment values given to the cells with crisp values in the corresponding transportation table, calculate the corresponding fuzzy cost.

3. Result and Discussion

3.1. Mathematical Explanation

This section has mentioned the mathematical explanation related to the above algorithm. Also, the results of other studies obtained by Yager's method are important to test the success of the proposed algorithm. Accordingly, the analytical comparison is illustrated by the graphs below.

Ex.1 [8]

Table 2. *Fuzzy transportation problem.*

	D_1	D_2	D_3	D_4	supply
S_1	(3,6,9)	(3,6,12)	(3,9,12)	(3,6,9)	(6,9,12)
S_2	(3,6,12)	(3,9,12)	(3,6,9)	(6,9,12)	(3,6,9)
S_3	(3,6,12)	(6,9,12)	(6,9,12)	(3,6,9)	(12,18,24)
S_4	(6,9,15)	(3,6,9)	(6,12,18)	(6,9,15)	(9,12,15)
demand	(15,18,21)	(6,9,12)	(3,9,12)	(6,9,15)	

Table 3. *Step 02.*

	D_1	D_2	D_3	D_4	supply
S_1	6	6.75	8.25	6	9
S_2	6.75	8.25	6	9	6
S_3	6.75	9	9	6	18
S_4	9.75	6	12	9.75	12
demand	18	9	8.25	9.75	

Consider table 3 crisp values are obtained by following formula

$$R(\tilde{C}_{11}) = R(3,6,9)$$

$$R(\tilde{a})=\int_0^1 0.5(a_\alpha^l \, , a_\alpha^u)d\alpha$$

Where:

$$a_\alpha^l = (6\text{-}3)\ \alpha+3 = 3\ \alpha + 3$$

$$a_\alpha^u = -(9\text{-}6)\ \alpha+9 = -3\ \alpha+9$$

$$\underset{\cdot}{R}(\tilde{a})=\int_0^1 0.5(a_\alpha^l\ , a_\alpha^u\ )d\alpha$$

$$\int_0^1 0.5(12)\ d\alpha = 6$$

Similarly

$$\underset{\cdot}{R}(\tilde{C}_{11}) =6,\ \underset{\cdot}{R}(\tilde{C}_{12}) =6.75,\ \underset{\cdot}{R}(\tilde{C}_{13})=8.25,\ \underset{\cdot}{R}(\tilde{C}_{14})=6$$

$$\underset{\cdot}{R}(\tilde{C}_{21}) =6.75,\ \underset{\cdot}{R}(\tilde{C}_{22}) =8.25,\ \underset{\cdot}{R}(\tilde{C}_{23})=6,\ \underset{\cdot}{R}(\tilde{C}_{24})=9$$

$$\underset{\cdot}{R}(\tilde{C}_{31}) =6.75,\ \underset{\cdot}{R}(\tilde{C}_{32}) =9,\ \underset{\cdot}{R}(\tilde{C}_{33})=9,\ \underset{\cdot}{R}(\tilde{C}_{34})=6$$

$$\underset{\cdot}{R}(\tilde{C}_{41}) =9.75,\ \underset{\cdot}{R}(\tilde{C}_{42}) =6,\ \underset{\cdot}{R}(\tilde{C}_{43})=12,\ \underset{\cdot}{R}(\tilde{C}_{44})=9.75$$

Table 4. *Step 03.*

	D_1	**D_2**	**D_3**	**D_4**	**supply**	**Average in each row $(A.R)_j$**
S_1	6	6.75	8.25	6	9	6.75
S_2	6.75	8.25	6	9	6	7.5
S_3	6.75	9	9	6	18	7.688
S_4	9.75	6	12	9.75	12	9.375
demand	18	9	8.25	9.75		
Average in each column $(A.C)_i$	7.313	7.5	8.813	7.688		

Consider table 4 Average crisp value in each row and column obtain by following formula

$$(A.R)_1 = \frac{\tilde{C}11+\tilde{C}12+\tilde{C}13+\tilde{C}14}{4} = 6.75$$

Similarly

$$(A.R)_1=6.75,\ (A.R)_2=7.5,\ (A.R)_3=7.688,\ (A.R)_4=9.375$$

$$(A.C)_1 =7.313,\ (A.C)_2=7.5,\ (A.C)_3=8.813,\ (A.C)_4=7.688$$

Table 5. *Step 04 & Step 05.*

	D_1	**D_2**	**D_3**	**D_4**	**supply**
S_1	1.032 $(9)^4$	0.375	-0.469	1.219	9 (0)
S_2	0.657	-0.75	2.157 $(6)^2$	-1.406	6 (0)
S_3	0.751 $(8.25)^5$	-1.406	-0.750	1.688 $(9.75)^3$	18 (8.25) (0)
S_4	-1.406 $(0.75)^6$	2.438 $(9)^1$	-2.906 $(2.25)^7$	-1.219	12 (3) (2.25) (0)
demand	18 (9) (0.75) (0)	9 (0)	8.25 (2.25) (0)	9.75 (0)	

$()^1, ()^2, ()^3$ order of the allocations (allocate order of the decreasing new value)

Consider the table 5 new values in each cell are obtain by following formula

$$V_{ij} = \frac{(A.R)\ _j+(A.C)_i}{2} - \tilde{C}_{ij}$$

$$V_{1,1} = \frac{(A.R)\ _1+(A.C)_1}{2} - \tilde{C}_{1,1}$$

$$V_{1,1} = \frac{6.75+7.313}{2} - 6 = 1.032$$

Similarly

P_{11}= 1.032, P_{12}= 0.375, P_{13}=-0.469, P_{14}=1.219
P_{21}=0.657, P_{22}=-0.75, P_{23}=2.157, P_{24}=-1.406
P_{31}= 0.751, P_{32}= -1.406, P_{33}=-0.750, P_{34}=1.688
P_{41}= -1.406, P_{42}=2.438, P_{43}=-2.906, P_{44}=-1.219

Table 6. *Step 06.*

	D_1	**D_2**	**D_3**	**D_4**	**supply**
S_1	6 (9)	6.75	8.25	6	9
S_2	6.75	8.25	6 (6)	9	6
S_3	6.75 (8.25)	9	9	6 (9.75)	18
S_4	9.75 (0.75)	6 (9)	12 (2.25)	9.75	12
demand	18	9	8.25	9.75	

Fuzzy transportation cost = $(6 \times 9) + (6.75 \times 8.25) + (9.75 \times 0.75) + (6 \times 9) + (6 \times 6) + (12 \times 2.25) + (6 \times 9.75) =$

292.5

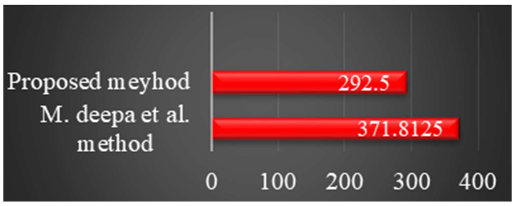

Figure 3. *Comparison with the proposed method and M. deepa et al. Method.*

Here are some more mathematical problems related to the explanations mentioned above. The related details are given as the grant steps are shown.

Ex.02 [15]

Table 7. *Fuzzy transportation problem.*

	D_1	**D_2**	**D_3**	**D_4**	**supply**
S_1	(2,3,3)	(2,3,3)	(2,3,3)	(1,4,4)	(0,3,3)
S_2	(4,9,9)	(4,8,8)	(2,5,5)	(1,4,4)	(2,13,13)
S_3	(2,7,7)	(0,5,5)	(0,5,5)	(4,8,8)	(2,8,8)
demand	(1,4,4)	(0,9,9)	(1,4,4)	(2,7,7)	

Table 8. *Step 02.*

	D_1	**D_2**	**D_3**	**D_4**	**supply**
S_1	2.75	2.75	2.75	3.25	2.25
S_2	7.75	7	4.25	3.25	10.25
S_3	5.75	3.75	3.75	7	(2,8,8)
demand	3.25	6.75	3.25	5.75	

Table 9. *Step 03.*

	D_1	**D_2**	**D_3**	**D_4**	**supply**	**Average in each row $(A.R)_j$**
S_1	2.75	2.75	2.75	3.25	2.25	2.875
S_2	7.75	7	4.25	3.25	10.25	5.563
S_3	5.75	3.75	3.75	7	6.5	5.063
demand	3.25	6.75	3.25	5.75		
Average in each column $(A.C)_i$	5.417	4.333	3.583	4.5		

Table 10. *Step 04 & Step 05.*

	D_1	**D_2**	**D_3**	**D_4**	**supply**
S_1	$1.396\ (2.25)^2$	0.854	0.479	0.438	2.25 (0)
S_2	$-2.26\ (1)^6$	$-2.052\ (0.25)^5$	$0.323\ (3.25)^4$	$1.782\ (5.75)^1$	10.25 (4.5) (1.25) (1) (0)
S_3	-0.51	$0.948\ (6.5)^3$	0.573	-2.219	6.5 (0)
demand	3.25 (1) (0)	6.75 (0.25) (0)	3.25 (0)	5.75 (0)	

Table 11. *Step 06.*

	D_1	**D_2**	**D_3**	**D_4**	**supply**
S_1	2.75 (2.25)	2.75	2.75	3.25	2.25
S_2	7.75 (1)	7 (0.25)	4.25 (3.25)	3.25 (5.75)	10.25
S_3	5.75	3.75 (6.5)	3.75	7	(2,8,8)
demand	3.25	6.75	3.25	5.75	

Fuzzy transportation cost $= (2.25 \times 2.75) + (7.75 \times 1) + (7 \times 0.25) + (3.75 \times 6.5) + (4.25 \times 3.25) + (3.25 \times 5.75) = 72.5625$

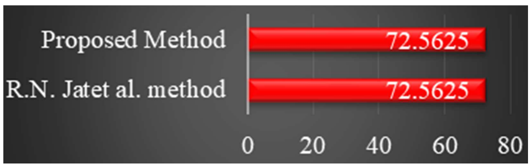

Figure 4. *Comparison with the proposed method and R. N. Jat et al. Method.*

Ex.03 [9]

Table 12. *Fuzzy transportation problem.*

	D_1	D_2	supply
S_1	(22,31,34)	(15,19,21)	(150,201,246)
S_2	(30,39,54)	(8,10,12)	(50,99,154)
demand	(100,150,200)	(100,150,250)	

Table 13. *Step 02.*

	D_1	D_2	supply
S_1	29.5	20.5	199.5
S_2	40.5	10	100.5
demand	150	150	

Table 14. *Step 03.*

	D_1	D_2	supply	Average in each row $(A.R)_j$
S_1	29.5	20.5	199.5	25
S_2	40.5	10	100.5	25.25
demand	150	150		
Average in each column $(A.C)_i$	35	15.25		

Table 15. *Step 04 & Step 05.*

	D_1	D_2	supply
S_1	0.5 $(150)^2$	-0.375 $(49.5)^3$	199.5 (49.5) (0)
S_2	-10.375	10.25 $(100.5)^1$	100.5 (0)
demand	150 (0)	150 (49.5)	

Table 16. *Step 06.*

	D_1	D_2	supply
S_1	29.5 (150)	20.5 (49.5)	199.5
S_2	40.5	10 (100.5)	100.5
demand	150	150	

Fuzzy transportation cost = $(29.5 \times 150) + (20.5 \times 49.5) + (10 \times 100.5) = 6444.75$

Ex.04 [10]

Table 17. *Fuzzy transportation problem.*

	D_1	D_2	D_3	supply
S_1	(15,25,35)	(55,65,85)	(85,95,105)	(75,95,125)
S_2	(65,75,85)	(80,90,110)	(30,40,50)	(45,65,95)
demand	(35,45,65)	(25,35,45)	(60,80,110)	

Table 18. *Step 02.*

	D_1	D_2	D_3	supply
S_1	25	67.5	95	97.5
S_2	75	92.5	40	67.5
demand	47.5	35	82.5	

Table 19. *Step 03.*

	D_1	D_2	D_3	supply	Average in each row $(A.R)_j$
S_1	25	67.5	95	97.5	62.5
S_2	75	92.5	40	67.5	69.169
demand	47.5	35	82.5		
Average in each column $(A.C)_i$	50	80	67.5		

Table 20. *Step 04 & Step 05.*

	D_1	D_2	D_3	supply
S_1	31.25 $(47.5)^1$	3.75 $(345)^3$	-30 $(15)^4$	97.5 (50) (15) (0)
S_2	-15.416	-17.916	28.335 $(67.5)^2$	67.5 (0)
demand	47.5 (0)	35 (0)	82.5 (15) (0)	

Table 21. *Step 06.*

	D_1	D_2	D_3	supply
S_1	25 (47.5)	67.5 (35)	95 (15)	97.5
S_2	75	92.5	40 (67.5)	67.5
demand	47.5	35	82.5	

Fuzzy transportation cost $=(25\times 47.5)+(67.5\times 35)+(95\times 15)+(40\times 67.5)=7675$

Ex.05 [7]

Table 22. *Fuzzy transportation problem.*

	D_1	D_2	D_3	D_4	supply
S_1	(2,4,6)	(2,4,8)	(2,6,8)	(2,4,6)	(4,6,8)
S_2	(2,4,8)	(2,6,8)	(2,6,4)	(4,6,8)	(2,4,6)
S_3	(2,4,8)	(4,6,8)	(4,6,8)	(2,4,6)	(8,12,16)
S_4	(4,6,12)	(2,4,6)	(4,8,12)	(4,6,10)	(6,8,10)
demand	(10,12,14)	(4,6,8)	(2,6,8)	(4,6,10)	

Table 23. *Step 02.*

	D_1	D_2	D_3	D_4	supply
S_1	4	4.5	5.5	4	6
S_2	4.5	5.5	4.5	6	4
S_3	4.5	6	6	4	12
S_4	6.5	4	8	6.5	8
demand	12	6	5.5	6.5	

Table 24. *Step 03.*

	D_1	D_2	D_3	D_4	supply	Average in each row $(A.R)_j$
S_1	4	4.5	5.5	4	6	4.5
S_2	4.5	5.5	4.5	6	4	5.125
S_3	4.5	6	6	4	12	5.125
S_4	6.5	4	8	6.5	8	6.25
demand	12	6	5.5	6.5		
Average in each column $(A.C)_i$	4.875	5	6	5.125		

Table 25. *Step 04 & Step 05.*

	D_1	D_2	D_3	D_4	supply
S_1	0.688 $(6)^4$	0.25	-0.25	0.813	6 (0)
S_2	0.5	-0.438	1.063 $(4)^3$	-0.875	4 (0)
S_3	0.5 $(5.5)^5$	-0.938	-0.438	1.125 $(6.5)^2$	12 (5.5) (0)
S_4	-0.938 $(0.5)^6$	1.625 $(6)^1$	-1.875 $(1.5)^7$	-0.813	8 (2) (1.5) (0)
demand	12 (6) (0.5) (0)	6 (0)	5.5 (1.5) (0)	6.5 (0)	

Table 26. *Step 06.*

	D_1	D_2	D_3	D_4	supply
S_1	4 (6)	4.5	5.5	4	6
S_2	4.5	5.5	4.5 (4)	6	4
S_3	4.5 (5.5)	6	6	4 (6.5)	12
S_4	6.5 (0.5)	4 (6)	8 (1.5)	6.5	8
demand	12	6	5.5	6.5	

Fuzzy transportation cost $= (4\times 6)+(4.5\times 5.5)+(6.5\times 0.5)+(4\times 6)+(4.5\times 4)+(8\times 1.5)+(4\times 6.5)=132$

Figure 5. *Comparison with the proposed method and P. Sagaya et al. Method.*

Ex.06 [18]

Table 27. *Fuzzy transportation problem.*

	D_1	D_2	D_3	supply
S_1	(1,4,9,19)	(1,2,5, 9)	(2,5,8,18)	(1,5,7,9)
S_2	(8,9,12,26)	(3,5,8,12)	(7,9,13,28)	(4,7,8,10)
S_3	(11,12,20,27)	(0,5,10,15)	(4,5, 8, 11)	(4,5,8,11)
demand	(3, 5, 8, 12)	(4, 8, 9,10)	(2, 4, 6, 8)	

Table 28. *Step 02.*

	D1	D2	D3	supply
S1	8.25	3.75	8.25	5.5
S2	13.75	7	14.25	7.25
S3	17.5	7.5	7	7
demand	7	7.75	5	

Table 29. *Step 03.*

	D1	D2	D3	supply	Average in each row $(A.R)_j$
S1	8.25	3.75	8.25	5.5	6.75
S2	13.75	7	14.25	7.25	11.667
S3	17.5	7.5	7	7	10.667
demand	7	7.75	5		
Average in each column $(A.C)_i$	13.167	6.083	9.833		

Table 30. *Step 04 & Step 05.*

	D1	D2	D3	supply
S1	1.709	2.667 $(5.5)^2$	0.042	5.5 (0)
S2	-1.333 $(5)^4$	1.875 $(2.25)^3$	-3.5	7.25 (5) (0)
S3	-5.583 $(2)^5$	0.875	3.25 $(5)^1$	7 (2) (0)
demand	7 (2) (0)	7.75 (2.25) (0)	5 (0)	

Table 31. *Step 06.*

	D1	D2	D3	supply
S1	8.25	3.75 (5.5)	8.25	5.5
S2	13.75 (5)	7 (2.25)	14.25	7.25
S3	17.5 (2)	7.5	7 (5)	7
demand	7	7.75	5	

Fuzzy transportation cost = $(13.75 \times 5) + (17.5 \times 2) + (3.75 \times 5.5) + (7 \times 2.25) + (7 \times 5) = 175.125$

Ex.07 [19]

Table 32. *Fuzzy transportation problem.*

	D_1	D_2	D_3	Supply
S_1	(3,5,7, 14)	(2,4,8, 13)	(3,5,9, 15)	35
S_2	(2,5,8, 10)	(3,6,9, 12)	(4,7,10,16)	40
S_3	(3,6,8, 13)	(4,8,10,15)	(5,9,13,15)	50
Demand	45	55	25	125

Table 33. *Step 02.*

	D_1	D_2	D_3	Supply
S_1	7.25	6.75	8	35
S_2	6.25	7.5	9.25	40
S_3	7.5	9.25	10.5	50
Demand	45	55	25	125

Table 34. *Step 03.*

	D_1	D_2	D_3	Supply	Average in each row $(A.R)_j$
S_1	7.25	6.75	8	35	7.333
S_2	6.25	7.5	9.25	40	7.666
S_3	7.5	9.25	10.5	50	9.083
Demand	45	55	25		
Average in each column $(A.C)_i$	7	7.833	9.25		

Table 35. *Step 04 & Step 05.*

	D_1	D_2	D_3	Supply
S_1	-0.083	8.416 (35)[1]	0.293	35 (0)
S_2	1.083 (20)[3]	8.000 (20)[2]	-0.792	40 (20) (0)
S_3	0.546 (25)[4]	7.667	-1.334 (25)[5]	50 (25) (0)
Demand	45 (25) (0)	55 (20) (0)	25 (0)	

Table 36. *Step 06.*

	D_1	D_2	D_3	Supply
S_1	7.25	6.75 (35)	8	35
S_2	6.25 (20)	7.5 (20)	9.25	40
S_3	7.5 (25)	9.25	10.5 (25)	50
Demand	45	55	25	125

Figure 6. *Comparison with the proposed method and D. Hunwisai et al. Method.*

Fuzzy transportation cost = $(6.25 \times 20) + (7.5 \times 25) + (6.75 \times 35) + (7.5 \times 20) + (10.5 \times 25) = 961.25$
Ex.08 [19]

Table 37. *Fuzzy transportation problem.*

	D_1	D_2	D_3	Supply
S_1	(2, 5, 8, 15)	(2, 3, 4, 7)	(3, 7, 9, 15)	25
S_2	(3, 6, 9, 12)	(4, 7, 9, 11)	(4, 8, 10, 13)	35
S_3	(3, 7, 10, 16)	(5, 6, 12, 16)	(4, 6, 8, 14)	50
S_4	(3, 4, 6, 9)	(4, 5, 7, 9)	(5, 8, 11, 13)	10
Demand	30	40	50	120

Table 38. *Step 02.*

	D_1	D_2	D_3	Supply
S_1	7.5	4	8.5	25
S_2	7.5	7.75	8.75	35
S_3	9	9.75	8	50
S_4	5.5	6.25	9.25	10
Demand	30	40	50	120

Table 39. *Step 03.*

	D_1	D_2	D_3	Supply	Average in each row $(A.R)_j$
S_1	7.5	4	8.5	25	6.667
S_2	7.5	7.75	8.75	35	8
S_3	9	9.75	8	50	8.917
S_4	5.5	6.25	9.25	10	7
Demand	30	40	50	120	
Average in each column $(A.C)_i$	7.375	9.938	8.625		

Table 40. *Step 04 & Step 05.*

	D_1	D_2	D_3	Supply
S_1	-0.479	4.303 (25)[1]	-0.854	25 (0)
S_2	0.188 (30)[5]	1.219 (5)[3]	-0.438	35 (30) (0)
S_3	-0.854	-0.323	0.771 (50)[4]	50 (0)
S_4	1.683	2.219 (10)[2]	-1.438	10 (0)
Demand	30 (0)	40 (15) (5) (0)	50 (0)	120

Table 41. *Step 06.*

	D_1	D_2	D_3	Supply
S_1	7.5	4 (25)	8.5	25
S_2	7.5 (30)	7.75 (5)	8.75	35
S_3	9	9.75	8 (50)	50
S_4	5.5	6.25 (10)	9.25	10
Demand	30	40	50	120

Fuzzy transportation cost = $(7.5 \times 30) + (4 \times 25) + (7.75 \times 5) + (6.25 \times 10) + (8.\times 50) = 821.25$

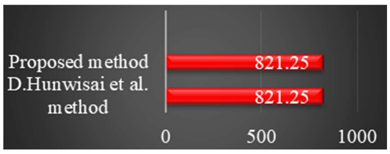

Figure 7. *Comparison with the proposed method and D. Hunwisai et al. Method.*

More mathematical problems related to those solved by using the proposed algorithm are figured in the following table. Also problems are compared with the existing methods, and the comparison graphs are as follow: here the problems are solved, and mention made of Appendix 1.

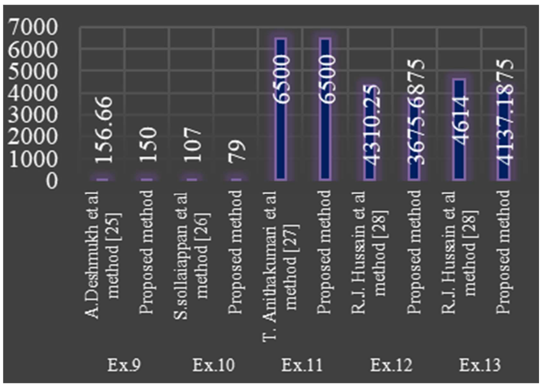

Figure 8. *The fuzzy transportation problems comparison with existing methods.*

Table 42. *The fuzzy transportation problems comparison with existing methods.*

Ex.9	A. Deshmukh et al method [25]	156.66
	Proposed method	150
Ex.10	S. sollaiappan et al method [12]	107
	Proposed method	79
Ex.11	T. Anithakumari et al method [26]	6500
	Proposed method	6500
Ex.12	R. J. Hussain et al method [27]	4310.25
	Proposed method	3675.6875
Ex.13	R. J. Hussain et al method [27]	4614
	Proposed method	4137.1875

As mentioned in the table above, the basic solution values obtained by solving several fuzzy transportation problems have been shown in comparison with each other method mentioned in the article's transportation problem. As a result, it has been shown that the basic solutions obtained by the proposed method are compared with other methods and provide successful solutions. The relevant graphical representation is shown below.

Below are some mathematical problems that were randomly determined using the algorithm mention above. The result obtain are compared with the optimal solution obtained by the Yager's robust ranking method with proposed method in the table below. Also, the analyzed chart shows the result more clearly. Details related to the relevant mathematical problems are mentioned in appendix 2.

Table 43. *Comparison with randomly determined problems optimum solution.*

Ex.14	Proposed method	300.9375
	Optimal solution	300.9375
Ex.15	Proposed method	441.5
	Optimal solution	441.5
Ex.16	Proposed method	598.125
	Optimal solution	545.62
Ex.17	Proposed method	411.38
	Optimal solution	411.38
Ex.18	Proposed method	627.75
	Optimal solution	627.75
Ex.19	Proposed method	78.75
	Optimal solution	77.12
Ex.20	Proposed method	1000
	Optimal solution	1000

In a transportation problem, when the basic solution is the same as the optimal solution, it can be pointed out that the method that obtains the relevant basic solution is a successful method. In it, the efficiency of the relevant algorithm has been demonstrated by being able to obtain optimal solutions with fewer steps and spending the least amount of time. Accordingly, the initial solutions are shown in comparison with the optimal solutions obtained by "Yager's ranking method" for eight randomly formulated fuzzy transportation problems. Optimal solutions could be obtained for five out of eight problems. It is clearly indicated by the graphical representation.

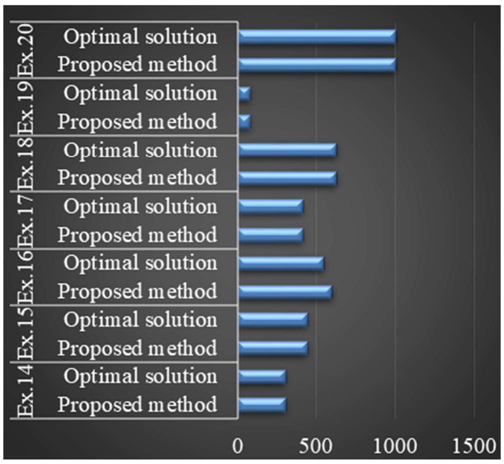

Figure 9. *Randomly determined problems comparison with optimum solution.*

3.2. Discussion

As presented in the mathematical explanation section above, the proposed algorithm solves the relevant transportation problems. There, the relevant steps for the first eight problems have been explained in detail, and the method of obtaining a basic solution has been shown. There, the answers obtained according to the proposed algorithm from the research article obtained from those problems and the answers obtained by the algorithm proposed in this paper have been shown in comparison with the graphs. It was found that the proposed algorithm gave more accurate answers. Meanwhile, the results of five more fuzzy transportation problems are contained in Table 42, and their comparative analysis is shown in Figure 8. The related problems are given in Table 44 in Appendix 1.

Algorithms are generally introduced with the goal of getting basic solutions to a transportation problem, and in this paper, an alternative algorithm has been prepared to get basic solutions for fuzzy transportation problems. The basic solutions obtained by using that algorithm, compared with the optimal solution obtained by Yager's "robust ranking method" for the related transportation problem, are shown in Table 44 and Figure 9. The related problems were randomly created, and the details related to those problems are mentioned in Table 45 of Appendix 2. According to the results obtained there, the initial value and optimal solution value were the same in five out of eight problems. And in the rest, the response was close to the optimal level. In this way, the composition of the test paper has been prepared through twenty fuzzy transportation problems. It can be shown by the relevant results that a successful algorithm was introduced to obtain basic solutions to fuzzy transportation problems, which was the aim of the research paper.

4. Conclusion

When considering transportation problems, there are different types of transportation models. Some categories are the most interesting in human life, where effective management is the main consideration. Through this study, we have proposed a new algorithm to find the basic solution to fuzzy transportation problems. In it, it is considered efficient to transport the goods from different sources to different destinations while minimizing the transportation cost. This objective can be pointed out as the basic characteristic of the traditional transportation problem. But it differs from the fuzzy transportation problem because the goods of transportation costs, supply, and demand have fuzzy values. The problem should be adjusted accordingly. Here we used Yager's Robust method and obtained an adjusted value according to the crisp values. Then, using the new values, the respective assignments were made. Also, the problems related to the existing methods have been checked, and the effectiveness of the proposed method has been shown analytically. Thus, as a final conclusion, this new algorithm that we introduce for fuzzy transportation problems can be shown as a method that provides an efficient basic solution that can be easily calculated.

Appendix

Table 44. *Analyzed results Table 42 contains the details of fuzzy costs, supplies, and demand for transportation problems.*

Example	$\tilde{C}_{ij}$	S_i	D_j
9 [25]	{[5,6,7], [2,3,4], [4,5,6,], [3,4,5]; [4,5,6], [8,9,10], [1,2,3], [6,7,8]; [4,5,6,], [6,7,8], [7,8,9], [5,6,7]}	{[21,22,23], [14,14,16], [6,8,10]}	{[6,7,8], [[11,12,13], [16,17,18], [8,9,10]}
10 [12]	{[-2,0,2,8], [-2,0,1,4], [-2.0,1,4], [-1,0,1,4]; [4,8,12,16], [4,7,9,12], [2,4,6,8], [[1,3,5,7]; [2,4,9,13], [0,6,8,10], [0,6,8,10], [4,7,9,12]}	{[0,2,4,6], [4,4,9,13], [2,4,6,8]}	{[1,3,5,7], [0,2,4,6], [1,3,5,7], [1,3,5,7]}
11 [26]	{[18,20,22], [28,30,32]; [6,10,14], [35,40,45]}	{[150,200,250], [80,100,120]}	{[100,150,200], [100,150,150]}
12 [27]	{[1,4,9], [16,25,36], [9,36,49]; [16,25,64], [36,64,81], [4,49,64]; [4,25,81], [23,36,64], [[49,64,81]	{[4,25,36], [16,36,49], [25,49,81]}	{[16,25,36], [16,36,49], [25,49,81]
13 [27]	{[1,4,9,16], [4,9,16,25], [[9,16,25,36], [16,25,36,49]; [4,9,16,25], [9,16,25,36], [16,25,36,49], [25,36,49,64]; [9,16,25,36], [[16,25,36,49], [25,36,49,64], [36,49,61,81]; [16,25,36,49], [25,36,49,64], [36,49,64,81]; [16,25,36,46], [25,36,49,64], [36,49,64,81], [25,36,49,81]}	{[16,25,36,49], [36,49,64,81], [25,36,49,64], [4,16,23,36]}	{[36,49,64,81], [25,36,49,64], [4,16,25,36], [16,25,36,49]}

Table 45. *Analyzed results Table 43 contains the details of fuzzy costs, supplies, and demand for transportation problems.*

Example	$\tilde{C}_{ij}$	S_i	D_j
Ex.14	{[5,10,13],[1,2,3],[4,6,8];[3,4,5],[1,5,6],[1,4,5];[3,6,9],[2,5,7],[1,4,5]}	{[16,30,45], [10,47,52], [3,18,58]}	{[11,16,51], [20,40,60], [15,30,45]}
Ex.15	{[2,4,6,8], [2,6,8,12], [8,10,12,14]; [6,8,10,12], [4,6,10,12], [8,10,12,14]; [2,10,112,14], [2,12,14,16], [8,10,12,14]}	{[16,20,24,24], [4,6,10,12], [10,20,24,34]}	{[10,1,16,16], [2,10,12,14], [24,28,28,30]
Ex.16	{[1,3,9], [4,5,6], [4,5,8]; [1,3,6], [3,5,7], [2,4,7]; [2,4,7], [4,9,10], [3,5,6]}	[20,40,60], [25,35,45], [30,50,60]	[25,35,45], [40,45,50], [35,40,55]
Ex.17	{ [3,7,11], [13,18,23], [6,13,20], [15,20,25]; [16,19,24], [3,5,7], [5,7,10], [20,23,26]; [11,14,17], [7,9,11], [2,3,4], [5,7,8]}	[13,16,20], [14,19,25], [15,18,20]	[6,8,10], [9,12,15], [11,13,16], [16,20,24]

Example	$\tilde{C}_{ij}$	S_i	D_j
Ex.18	{[5,20,22,23], [7,29,30,31], [4,20,22,24]; [2,15,17,18] [2,16,18,20], [7,33,34,36]; [4,21,23,25], [5,22,23,25], [6,16,19,21]}	[10,12,14,20], [4,6,8,24], [10,14,16,20]	[6,8,10,12], [2,4,18,22], [16,18,20,22]
Ex.19	{[1,2,34], [1,3,4,6], [8,9,11,13], [5,6,8,9]; [0,1,2,4], [2,3,5,6], [5,6,7,8], [0,1,2,4]; [3,4,5,6], [12,14,16,17], [7,8,9,11] [7,8,10,13]	[2,6,8,9], [4,7,9,11], [4,5,7,9]	[5,7,8,9], [2,6,7,9], [1,3,4,5], [1,3,4,7]
Ex.20	{[2,4,7,9], [[2,3,4,7], [7,8,9,9], [1,2,3,4]; [5,6,7,9], [2,4,6,8], [5,8,11,13], [3,3,4,6]; [4,8,10,11], [5,6,12,13], [4,5,7,9], [5,8,10,11]; [3,5,8,9], [4,5,7,9], [4,7,9,12], [5,8,10,13]}	[10,30,60,100], [35,45,55,65], [30,40,60,70], [44,50,50,56]	[46,48,52,54], [40,50,50,60], [45,48,52,55], [34,48,52,66]

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